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# Thermodynamic optimization of free convection film condensation on a horizontal elliptical tube with variable wall temperature

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## Abstract

This study focuses on the thermodynamic analysis of saturated vapor flowing slowly onto and condensing on an elliptical tube with variable wall temperature. An entropy generation minimization (EGM), technique is applied as a unique measure to study the thermodynamic losses caused by heat transfer and film-flow friction for the laminar film condensation on a non-isothermal horizontal elliptical tube. The results provide us how the geometric parameter ellipticity and the amplitude of non-isothermal wall temperature variation affect entropy generation during filmwise condensation heat-transfer process. The optimal design can be achieved by analyzing entropy generation in film condensation on a horizontal elliptical tube with further account for the amplitude of non-isothermal wall temperature variation.

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# 1. Introduction

Filmwise condensation heat transfer of pure vapor flowing onto a body, such as a plate, tube, and sphere has been widely studied by many researchers, like Winkler et al. [1], and Yang and Hsu [2], in view of the practical importance in the design of condensers for power plants, air-conditioning equipments, and many other chemical industrial process equipments. Entropy generation in thermal engineering systems destroys energy available in the system, and reduces its efficiency, such as condenser and heat exchanger. Thus, entropy generation minimization is of great concern in phase-change heat-transfer problems associated with film condensation.

Bejan [3] pioneered the method of entropy generation minimization in heat and mass transfer analysis. Meanwhile, in his book [4], he also conducted the second-law

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analysis of thermodynamics via the minimization of entropy generation for the single phase convection heat transfer. Jani [5] provided optimization of falling film LiBr solution on a horizontal single tube based on the minimization of entropy generation that irreversibility of non-isothermal heat transfer dominates in comparison with the fluid flow friction and mass transfer. Sahin [6] investigated the effect of temperature-dependent viscosity on the entropy generation rate as well as the ratio of pumping power to heat transfer.

In a study on the forced-convection cooling of an electronic device consisting of a stack of printed circuit boards with heat generation chips, Furukawa [7] employed the entropy generation minimization (EGM) method to determine the optimum board spacing to maximize heat dissipation. The accuracy and reliability of the EGM method were confirmed by a satisfactory agreement between its predicted optimal board spacing and that obtained by the convectional thermal optimization method. Furukawa and Yang used the EGM method to optimize the fin pitch of a plate fin heat sink in free convection [8] and the channel

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# Nomenclature

A	amplitude of non-isothermal wall temperature	$\Delta T$	$T_{\rm sat} - T_{\rm w}$
	variation	$\Delta T$	$T_{\rm sat} - \overline{T_{\rm w}}$
a	semi-major axis of ellipse	и	velocity component in x direction
b	semi-minor axis of ellipse	v	velocity component in y direction
$Bo(\phi)$	$\phi$ -varying Bond number as defined by Eq. (6)		
Bo	Bond number, $(\rho - \rho_v)ga^2/\sigma$	Greek symbols	
Br	Brinkman number, $\mu u_0^2 / k \overline{\Delta T}$	$\delta$	thickness of condensate film
$C_{\rm p}$	specific heat capacity of condensate	$\delta^*$	dimensionless thickness of condensate film, de-
$D_{\rm e}$	equivalent diameter of elliptical tube		fined in Eq. (17)
е	ellipticity of ellipse	$\theta$	angle measured from the top of the tube
$F_{\rm t}(\phi)$	non-isothermal wall temperature variation	μ	absolute viscosity of condensate
g	acceleration due to gravity	ho	density of condensate
h	condensing heat-transfer coefficient at angle $\phi$	$ ho_{ m v}$	density of vapor
$h'_{fg}$	latent heat of condensation corrected for con-	$\sigma$	surface tension coefficient in the film
	densate subcooling	$\phi$	angle between the tangent to tube surface and
Ja	Jakob number, $C_{\rm p}(T_{\rm sat} - \overline{T_{\rm w}})/h'_{fg}$		the normal to direction of gravity
k	thermal conductivity of condensate	$\varphi$	the irreversibility distribution ratio, defined in
'n	condensate mass-flow rate per unit length of		Eq. (32)
	elliptical tube	$\Theta$	dimensionless temperature difference, $\overline{\Delta T}/T_{sat}$
$N_{\rm F}$	film flow friction irreversibility		
$N_{\rm H}$	heat-transfer irreversibility	Subscripts	
$N_{\mathbf{S}}$	the entropy generation number	opt	optimal
Nu	local Nusselt number, $hD_{\rm e}/k$	sat	saturation
$S''_{\rm gen}$	the volumetric entropy generation rate, defined	v	vapor
Ū.	in Eq. (21)	W	tube wall
$S_{\mathrm{o}}$	characteristic transfer rate		
Ra	Rayleigh number, $(\rho - \rho_v)\rho g Pr D_e^3/\mu^2$	Superscripts	
$T_{\rm sat}$	saturate temperature of vapor	0	reference value
$T_{\rm w}$	wall temperature	<b>т</b>	indicates dimensionless

flow in a package of parallel boards with discrete block heat sources [9].

From the above studies, one may see that entropy generation is associated with thermodynamic irreversibility which is common in all types of heat-transfer processes. Film condensation belongs to phase-change heat transfer, but little literature regarding its second-law analysis is investigated. The second-law analysis of the film condensation outside tubes still remains an unsettled question so far.

Adeyinka and Naterer [10] investigated the physical significance of entropy generation in plate film condensation. Their results for optimizing entropy generation and plate size are expressed in terms of a duty parameter. In addition, they observed that entropy generation provides a useful parameter in the optimization of a two-phase system. Lin et al. [11] discussed the second-law analysis on saturated vapor flowing through and condensed in horizontal cooling tubes. They noted that an optimum Reynolds number existed over the parametric range which the entropy generates at a minimum rate. Dung and Yang [12] used the EGM technique to conduct the second-law analysis in a saturated vapor flowing slowly onto and condensing on an isothermal horizontal tube and obtained an optimal

diameter that generates a minimum of entropy at a given duty.

As for the enhancement of condensation heat transfer, several researches, such as Yang and Hsu [13] and Yang and Chen [14], Ali and McDonald [15] and Karimi [16] confirmed that tubes, fins, or extended surfaces of elliptical profiles with major axes aligned with gravity are superior to those of circular profiles. In addition to heat-transfer analysis, Li and Yang [17] started to conduct the thermodynamic analysis of saturated vapor flowing slowly onto and condensed on an isothermal elliptical tube. That paper investigated how the geometric parameter-ellipticity affects local entropy generation rate during filmwise condensation heat transfer process. On the other hand, Fujii et al. [18] presented that the wall temperature may often vary significantly over the circumferential length of the tube, even if the condensation on a circular tube with a variable wall temperature (a cosine distribution). For laminar filmwise condensation with vapor flow velocity and inclusion of pressure gradient effect, the mean heat-transfer coefficient is influenced significantly with increasing the wall temperature variation amplitude, as seen in Memory and Rose [19].

The present study will focus on the minimization of total entropy generation number to give an idea of optimal design on free convection film condensation outside an elliptical tube with variable wall temperature. An expression for the entropy generation number accounts for the combined action of the specified irreversibilities. This research on the entropy generation minimization will thus help us achieve the complete thermodynamic analysis, including first and second law, on laminar filmwise condensation outside a non-isothermal elliptical tube.

## 2. Thermal analysis

Consider a horizontal elliptical tube with major axis "2a" in the gravitational direction and minor axis "2b", situated in a quiescent pure vapor which is at its saturated temperature  $T_{\text{sat}}$ . Moreover, the wall temperature  $T_{\text{w}}$  is considered to be non-uniform and much lower than the vapor saturation temperature  $T_{\text{sat}}$ . Thus, condensation occurs on the wall and a continuous film of the liquid runs downward over the tube under the influence of gravity.

The physical model under consideration is shown in Fig. 1 where the curvilinear coordinates (x, y) are aligned along the elliptical wall surface and its normal. The assumptions employed in the formulation of the problem are

- (1) The condensate film flow is laminar and steady-state.
- (2) The inertia effect is neglected.
- (3) Viscous dissipation is ignored.
- (4) Compared with the transversal conduction, the axial conduction is negligible.
- (5) The condensate film thickness is much smaller than the curvature diameter.

According to the above assumptions, the condensate film governed equations are written as follows:



Fig. 1. Physical model and coordinate system for condensate film flow on an elliptical surface.

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{1}$$

Momentum equation:

$$\mu \frac{\partial^2 u}{\partial y^2} = -(\rho - \rho_v)g[\sin \phi + Bo(\phi)].$$
<sup>(2)</sup>

Energy equation:

$$k\frac{\partial^2 T}{\partial y^2} = 0.$$
(3)

It is further assumed that at the interface, no vapor shear is considered to exert upon the condensate. Thus, the boundary conditions are

$$y = 0; \quad u = 0; \quad T = T_{w};$$
 (4)

$$y = \delta; \quad \frac{\partial u}{\partial y} = 0; \quad T = T_{\text{sat}}.$$
 (5)

On account of varying radius of surface curvature, the surface tension forces can be derived here, as expressed in Yang and Chen [14]:

$$Bo(\phi) = \pm \frac{1}{Bo} \frac{3e^2\sigma}{2a^2} \left(\frac{1 - e^2\sin^2\phi}{1 - e^2}\right)^2 \sin(2\phi).$$
(6)

Integrating Eqs. (2) and (3) directly with the boundary conditions gives the following formula of the film velocity "u" and temperature "T" profile, respectively.

$$u = \frac{(\rho - \rho_{\nu})}{\mu} g \delta^2 [\sin \phi + Bo(\phi)] \left[ \frac{y}{\delta} - \frac{1}{2} \left( \frac{y}{\delta} \right)^2 \right],\tag{7}$$

$$T = \Delta T \cdot \frac{y}{\delta} + T_{\rm w},\tag{8}$$

where  $\Delta T = T_{\text{sat}} - T_{\text{w}}$ .

By assuming a reference velocity,

$$u_0 = \frac{(\rho - \rho_v)gD_{\rm e}^2}{2\mu}.$$
(9)

Eq. (7) becomes

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$$u(y) = u_0 (2y\delta - y^2) [\sin(\phi) + Bo(\phi)] / D_e^2.$$
(10)

Let  $\dot{m}$  be the mass-flow rate over an elliptical perimeter per unit tube length, and  $h'_{fg} = h_{fg}(1 + 0.68C_p\Delta T/h_{fg})$ , latent heat of condensation corrected for condensate subcooling by Rohsenow [20]. Utilizing equation (7), one obtains the local rate of the condensate mass flow per unit tube length as follows:

$$\dot{m} = \rho(\rho - \rho_{\rm v}) \frac{g\delta^3}{3\mu} [\sin\phi + Bo(\phi)].$$
(11)

An energy balance on an element of the condensate film of height  $\delta$  and width dx is

$$\frac{\mathrm{d}\dot{m}}{\mathrm{d}x} = \frac{k\Delta T}{h'_{fg}\delta}.$$
(12)

In order to derive the local film thickness  $\delta$  at the circumferential arc length x in terms of  $\phi$ , one can substitute Eq. (11) into Eq. (12) and obtain

$$\frac{\rho(\rho - \rho_{\rm v})}{3\mu}gh'_{fg}\frac{\mathrm{d}}{\mathrm{d}x}(1 - \mathrm{e}^{2}\sin^{2}\phi)^{3/2}\frac{\mathrm{d}}{\mathrm{d}\phi}\{\delta^{3}[\sin\phi + Bo(\phi)]\}$$
$$= \frac{k\Delta T}{\delta}.$$
(13)

Once the wall temperature distribution  $T_w(\phi)$  is specified or fitted by experimental data, the mean wall temperature is really available as

$$\overline{T_{\rm w}} = \frac{2a}{\pi D_{\rm e}} \int_0^{\pi} T_{\rm w}(\phi) [(1 - {\rm e}^2) / \sqrt{(1 - {\rm e}^2 \sin^2 \phi)^3}] \,\mathrm{d}\phi, \quad (14)$$

and subsequently the temperature difference across the film can be expressed as

$$T_{\rm sat} - T_{\rm w} = \overline{\Delta T} F_{\rm t}(\phi), \tag{15}$$

where  $\overline{\Delta T} = T_{\text{sat}} - \overline{T_{\text{w}}}$ . Representative numerical results for the common axisymmetric case that involves the cosine distribution of non-isothermal wall temperature variation are given as

$$F_{t}(\phi) = 1 - A\cos(\phi). \tag{16}$$

Here, the non-isothermality function is adopted from the experiment of Lee et al. [21] for circular tube. Note that  $0 \le A \le 1$  and the amplitude *A* largely depends on the ratio of the outside-to-inside heat-transfer coefficients.

Using separation of variables, one may derive dimensionless local condensate liquid film thickness as

$$\delta^{*} = \delta \left[ \frac{D_{e} k \mu \overline{\Delta T}}{g h'_{fg} \rho (\rho - \rho_{v})} \right]^{-1/4}$$

$$= \left[ \sin \phi + Bo(\phi) \right]^{\frac{-1}{3}} \frac{\left\{ 2(1 - e^{2}) \int_{0}^{\phi} F_{t}(\phi) \frac{\left[ \sin \phi + Bo(\phi) \right]^{\frac{1}{3}}}{(1 - e^{2} \sin^{2} \phi)^{\frac{3}{2}}} d\phi \right\}^{\frac{1}{4}}}{\left\{ \frac{1}{\pi} \int_{0}^{\pi} \left[ \frac{(1 - e^{2})}{\sqrt{(1 - e^{2} \sin^{2} \phi)^{3}}} d\phi \right] \right\}^{\frac{1}{4}}}.$$
(17)

As in Nusselt [22] theory, interpreting a local heat-transfer coefficient gives

$$Nu = \frac{hD_{\rm e}}{k} = \frac{\left[\frac{Ra}{J_a}\right]^{\frac{1}{4}}}{\delta^*},\tag{18}$$

where,

$$\begin{aligned} Ra &\equiv \frac{\rho(\rho - \rho_{\rm v})g\,PrD_{\rm e}^3}{\mu^2}\\ Ja &\equiv \frac{C_{\rm p}\overline{\Delta T}}{h'_{fg}}. \end{aligned}$$

According to Bejan [3], together with the fifth item of the above-mentioned assumptions, the entropy generation rate for convection heat transfer can be written as

$$S_{\text{gen}}^{\prime\prime\prime} = \frac{k}{T^2} \left(\frac{\partial T}{\partial y}\right)^2 + \frac{\mu}{T} \left(\frac{\partial u}{\partial y}\right)^2.$$
(19)

On the right-hand side of Eq. (19), the first term and the second term represent the entropy generation due to heat transfer and due to film flow friction, respectively.

Assuming that the temperature difference between the saturated temperature and condensate film is much smaller than the temperature of condensate film yields

$$T \approx T_{\rm sat}.$$
 (20)

Substituting Eqs. (8), (10), (15) and (20) into Eq. (19) we obtain

$$S_{\text{gen}}^{\prime\prime} = \frac{k}{T_{\text{sat}}^2} \left(\frac{\Delta T F_{\text{t}}(\phi)}{\delta}\right)^2 + \frac{\mu}{T_{\text{sat}}} \left(\frac{u_0(2\delta - 2y)[\sin(\phi) + B_{\text{o}}(\phi)]}{D_{\text{e}}^2}\right)^2.$$
(21)

Integrating Eq. (21) with respect to y from zero to  $\delta$  yields

$$S'_{\text{gen}} = \frac{k}{T_{\text{sat}}^2} \left(\frac{\overline{\Delta T} F_{\text{t}}(\phi)}{\delta}\right)^2 \delta + \frac{4\mu}{3T_{\text{sat}}} \left\{\frac{u_0[\sin(\phi) + Bo(\phi)]}{D_{\text{e}}^2}\right\}^2 \delta^3.$$
(22)

Next, integrating Eq. (22) over the entire streamline length, from  $\phi = 0$  to  $\pi$  gives

$$S_{\rm gen} = \frac{k(\overline{\Delta T})^2}{2T_{\rm sat}^2} (Ra/Ja)^{1/4} I_{\rm r} + \frac{2u_0^2\mu}{3T_{\rm sat}} (Ra/Ja)^{-3/4} I_{\rm d},$$
(23)

where,

$$I_{\rm r} = \int_0^{\pi} \frac{\left(F_{\rm t}\right)^2}{\delta^*} \,\mathrm{d}\phi \quad \text{and}$$
  

$$I_{\rm d} = \int_0^{\pi} \left(\delta^*\right)^3 \left[\sin(\phi) + Bo(\phi)\right]^2 \,\mathrm{d}\phi, \qquad (24)$$

where k,  $T_{\text{sat}}$  and  $\mu$  denote thermal conductivity, saturated temperature, and dynamic viscosity, respectively. Entropy generation number ( $N_{\text{S}}$ ) is the ratio of the volumetric entropy generation rate ( $S_{\text{gen}}$ ) to a characteristics transfer rate ( $S_{\text{o}}$ ).

$$N_{\rm S} = \frac{S_{\rm gen}}{S_{\rm o}},\tag{25}$$

where,

$$S_{\rm o} = \frac{k(\overline{\Delta T})^2}{T_{\rm sat}^2}.$$
 (26)

Further, by introducing the following dimensionless parameters

$$Br = \mu \frac{u_0^2}{k\Delta T},\tag{27}$$

$$\Theta = \frac{\overline{\Delta T}}{T_{\text{sat}}},\tag{28}$$

the entropy generation number can be expressed as

$$N_{\rm s} = \frac{1}{2} (Ra/Ja)^{1/4} I_{\rm r} + \frac{2}{3} \frac{Br}{\Theta} (Ra/Ja)^{-3/4} I_{\rm d}$$
  
=  $N_{\rm H} + N_{\rm F}.$  (29)

To understand as to which of the condensate flow friction irreversibility  $(N_{\rm F})$ , or heat-transfer irreversibility  $(N_{\rm H})$  dominates, we introduce a criterion known as the irreversibility distribution ratio in the following equation:

$$\varphi = \frac{N_{\rm F}}{N_{\rm H}}.\tag{30}$$

Setting  $\frac{\partial N_S}{\partial (Ra/Ja)} = 0$ , we find the following optimum that minimizes the value of  $N_S$ 

$$(Ra/Ja)_{\rm opt} = 4\frac{I_{\rm d}}{I_{\rm r}}\frac{Br}{\Theta}.$$
(31)

Inserting Eq. (31) into Eqs. (23) and (25) gives an expression of minimizing entropy generation as follows:

$$(N_{\rm S})_{\rm opt} = (Ra/Ja)_{\rm opt}^{\frac{1}{4}} \frac{2}{3} I_{\rm r}.$$
 (32)

The ratio of the actual entropy generation to the minimized entropy generation represents  $N_{\rm S}^*$ , which is determined to be

$$N_{s}^{*} = \frac{N_{s}}{(N_{s})_{opt}} = \frac{\frac{1}{2}(Ra/Ja)^{\frac{1}{4}}I_{r}}{\frac{2}{3}(Ra/Ja)^{\frac{1}{4}}_{opt}I_{r}} + \frac{\frac{2}{3}(Ra/Ja)^{\frac{-2}{4}}\frac{Br}{\Theta}I_{d}}{\frac{2}{3}(Ra/Ja)^{\frac{1}{4}}_{opt}I_{r}}$$
$$= \frac{3}{4} \left[ \frac{Ra/Ja}{(Ra/Ja)_{opt}} \right]^{\frac{1}{4}} + \frac{1}{4} \left[ \frac{(Ra/Ja)_{opt}}{Ra/Ja} \right]^{\frac{3}{4}}.$$
(33)

#### 3. Results and discussion

The variation of dimensionless entropy generation numbers  $N_{\rm S}$  with Ra/Ja under the surface tension effects and variable wall temperature for various ellipticities are demonstrated in Fig. 2. Firstly, as in Yang and Chen [14] study, the present result also indicates that the mean heat-transfer coefficients increase with ellipticity. Secondly, the dimensionless entropy generation numbers augment with mean heat-transfer coefficients, i.e.,  $N_{\rm S}$  is proportional to Ra/Ja. Hence, the dimensionless entropy generation number increases with an increase in the ellipticity and Ra/Ja. Finally, the entropy generation number is nearly unaffected by surface tension forces at a small ellipticity such as  $e \leq 0.7$ , but somewhat influenced at a large ellipticity for whole perimeters. Namely, the effect of surface tension on the entropy generation number is significant at a larger ellipticity.

Fig. 3 shows that the variation of dimensionless entropy generation numbers  $N_{\rm S}$  with Ra/Ja under the effects of non-isothermal wall temperature variation and various ellipticities. The entropy generation number is markedly affected by the variable wall temperature at A = 1. This may account for the larger temperature differences.



Fig. 2. The variation of dimensionless entropy generation numbers  $N_{\rm S}$  with surface tension effect and various ellipticities versus Ra/Ja.



Fig. 3. The variation of dimensionless entropy generation numbers  $N_S$  with amplitude of non-isothermal wall temperature variation and various ellipticities versus Ra/Ja.

Fig. 4 indicates the minimum entropy generation rate versus  $Br/\Theta$  for two different values of amplitude of nonisothermal wall temperature variation, A. It is clear from Fig. 4 that the minimum entropy generation rate increases with amplitude of non-isothermal wall temperature variation and ellipticities. This may account for the fact that the optimal (Ra/Ja) is proportional to  $Br/\Theta$ , as seen in Eq. (31).

In Fig. 5,  $N_{\rm H}$ ,  $N_{\rm F}$  and  $N_{\rm S}$  versus Ra/Ja are drawn for several different values of ellipticities, respectively. Eq. (29) shows that the total entropy generation number is induced by the heat transfer irreversibility,  $N_{\rm H}$  and film flow friction irreversibility,  $N_{\rm F}$ . Apparently,  $N_{\rm H}$  varies as  $(Ra/Ja)^{1/4}$  and  $N_{\rm F}$  varies as  $(Ra/Ja)^{-3/4}$ . It is obvious that the contribution to entropy generation rate caused by heat transfer is much more than by the film flow friction.



Fig. 4. Minimum entropy generation rate versus  $Br/\Theta$ .



Fig. 5. Dimensionless entropy generation number versus Ra/Ja for various ellipticities.

In general, for the convection heat-transfer problem, both fluid friction and finite temperature difference heat transfer contribute to the rate of entropy generation. The irreversibility distribution ratio  $\varphi$  takes care of which irreversibility had dominated. The entropy generation rate is dominated by film flow friction irreversibility when  $\varphi > 1$ ; while the entropy generation rate is dominated by heat-transfer irreversibility when  $\varphi < 1$ . So, this may account for the finite temperature difference heat transfer across the condensate film thickness in Fig. 6. Note that irreversibility distribution ratio drops as ellipticities increases, namely, heat-transfer irreversibility plays a more important role for an elliptical tube with higher ellipticity.

Fig. 7 presents the variation of irreversibility as a function of the Ra/Ja for amplitude of non-isothermal wall temperature variation, A, and ellipticities. The irreversibility distribution ratio for the case A = 0 is larger than that for the case A = 1. This may account for the more contribution to irreversibility from larger temperature difference heat transfer.



Fig. 6. The irreversibility distribution ratio versus Ra/Ja for various ellipticities.



Fig. 7. The irreversibility distribution ratio with amplitude of nonisothermal wall temperature variation and various ellipticities versus Ra/Ja.

Next, minimum entropy generation rate versus ellipticities in Fig. 8 demonstrates that total dimensionless entropy generation numbers increase with  $Br/\Theta$  and ellipticities. It follows from this result that amplitude of non-isothermal wall temperature variation, ellipticities and  $Br/\Theta$  do cause the increase of minimum entropy generation rate. Notably, the amplitude of non-isothermal wall temperature variation plays a significant role in the growth of minimum entropy generation rate, for instance, A = 1. Meanwhile, for isothermal wall case, A = 0, the analysis also agreed with Li and Yang [23] in the fact that the optimal entropy generation number is proportional to the four root of group parameter, Ra/Ja. Therefore, the amplitude of non-isothermal wall temperature variation is the major concern for the second law based on minimization of total entropy generation rate.



Fig. 8. Minimum entropy generation rate versus ellipticities.

#### 4. Concluding remarks

An analytical study was performed on the entropy generation minimization of free convection film condensation on a non-isothermal horizontal elliptical tube. The result indicated that the optimal entropy generation number is proportional to one-fourth power of group parameter, Ra/Ja and to amplitude of non-isothermal wall temperature variation. The obtained results apply to quiescent vapor condensed outside horizontal elliptical tubes, and to very long elliptical tubes, with negligible interfacial vapor shear drag and variable wall temperature. The optimal design can be achieved by analyzing entropy generation in film condensation on a horizontal elliptical tube; however, the practical ellipticity is limited to 0.9 owing to manufacturing availability. Notably, since the condensate film temperature is always smaller than the saturated temperature, the assumption of Eq. (20) for simplicity will make the entropy generation rate of Eq. (21) become smaller than that of Eq. (19). Further, owing to negligible streamwised (i.e., x-directional) conduction, the heat transfer irreversibility due to finite temperature difference is also underestimated. For a practical interest, the present analysis on film condensation of a horizontal elliptical tube can apply to that of an inclined tube since a gravitational plane passing through an inclined circular tube yields an ellipse.

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